1. [10%] The austenite state of steel with 750°C is quenched with 200°C media by the following Newton's cooling law. During the first 30 minutes, the metal is cooled down to 300°C

\[
\frac{dT}{dt} = k(T - 200)
\]

where \( T \) is temperature of the metal, \( t \) is the quench time, and \( k \) is the proportional constant.

Determine (1) the constant \( k \) (1/min)

(2) the temperature after the first 60 minutes.

2. [15%] Solve the given initial-value problem

\[
\begin{align*}
\frac{dx}{dt} &= 2x - 3y \\
\frac{dy}{dt} &= x - 1
\end{align*}
\]

\( x(0) = 0, y(0) = 0 \)

3. [10%] Matrix

\[
A = \begin{bmatrix}
6 & 3 \\
-10 & -5
\end{bmatrix}
\]

Determine (1) the eigenvalues and eigenvectors

(2) \( A^{10} \)

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4. [10%] Solve the given initial-value problem

\[ y'' + 4y = \delta(t - \pi) \quad y(0) = 0, \quad y'(0) = 2 \]

where \( \delta(t - \pi) \) is an impulse function.

5. [10%] A linear system \( Ax = B \)

\[
A = \begin{bmatrix} 4 & 1 & 3 \\ 1 & 3 & -2 \\ 2 & -5 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ -7 \\ 19 \end{bmatrix}
\]

Determine (1) the rank of matrix \( A \)
(2) solve the unknowns
(3) sketch the solution in Cartesian coordinates

6. [10%] The homogeneous circular disk of mass \( m \), is located in the \( x-y \) plane and described by \( x^2 + y^2 = R^2 \).
Find the mass moment of inertia (1) \( I_x \) (2) \( I_z \)

7. [10%] The three components of velocity in a flow field are given by
\[ u = x^2 + y^2 + z^2, \quad v = xy + yz + z^2, \quad \text{and} \quad w = -3xz - (z^2/2) + 4. \]
Determine (1) the volumetric dilatation
(2) the rotation velocity.

8. [10%] A plane contains the given points (1, 3, -1), (0, 1, 0), (0, 1, 1),
Determine (1) the normal vector
(2) the equation of the plane
9. [15%] A torque is applied to the free end of a circular shaft and suddenly removed. The shaft is elastic rotating deformation with fixed end at \( x = 0 \), and free at \( x = 1 \). Via nondimensionalizing, the governing equation is given by

\[
\frac{\partial^2 \theta}{\partial x^2} = \frac{\partial^2 \theta}{\partial t^2}
\]

where \( \theta \) is the vibrating rotation angle, \( t \) is the time variable.

The initial and boundary conditions are

\[
\theta(0, t) = 0 \quad \frac{\partial \theta}{\partial x}(1, t) = 0
\]

\[
\theta(x, 0) = \theta_0 x \quad \frac{\partial \theta}{\partial t}(x, 0) = 0
\]

Determine (1) the vibrating mode shape function for the shaft.

(2) the \( \theta(x, t) \) at the free end \( (x = 1) \)