1. [15%] Consider the driven mechanical oscillator governed by the differential equation

\[ m\ddot{x} + c\dot{x} + kx = F(t) \]

where \( m = 0.4\text{kg}, c = 5N - s/m, k = 1400N/m \), and let \( F(t) \) is a periodic excitation force

\[ F(t) = \begin{cases} 2 & 0 < t < \pi \\ -2 & \pi < t < 2\pi \end{cases} \quad F(t + 2\pi) = F(t) \]

Find the steady-state oscillation?

2. [15%] (a) Draw a labeled sketch of the graph of the function \( f(t) \), where \( H(t) \) is a Heaviside function (unit step function).

(b) Find the Laplace transform of \( f(t) \)? \( f(t) = H(t - \pi)\cos(t) \)

3. [10%] Solve \( y(t) \) in the equation

\[ y = t - 9 \int_0^t y(\tau)(t - \tau)d\tau \]

4. [15%] The one dimensional heat equation subject to the given conditions

\[ u(0, t) = 0 \quad u(L, t) = 0 \]

\[ u(x, t) = \begin{cases} \pi/2 & 0 < x < L/2 \\ 0 & L/2 < x < L \end{cases} \]

(a) Write out the differential equation (b) Solve the temperature function \( u(x, t) \)?
5. [10%] Solve the differential equation

\[ \frac{d^2u}{dx^2} + \pi^2 u - 1 = 0 \quad 0 < x < 1 \]

with boundary conditions

\[ u(0) = 0 \quad u(1) = 0 \]

6. [10%] Determine a formula for the \( k \)-th power of the matrix

\[ A = \begin{bmatrix} 7 & -1 \\ 6 & 2 \end{bmatrix} \]

7. [15%] Find the general solution to the differential equation

\[ \frac{dy}{dx} = \frac{2x + y}{2y - x} \]

8. [10%] Find the directional derivative of \( W = x^2y + 2xy^2 - z^3 \) at \( (1, 2, 3) \) in the direction \( d = i - 2j + 2k \)