predictive control part and a disturbance compensation part. The steady-state performance due to the inaccuracy of the motor parameters can be obtained from eqns. 7 and 1. In the case of predictive control without feedforward compensation of \( f_a \), the current error at the \((k+1)\)th instant can be derived as \( i_a(k+1) - i_a(k+1) = (T/L_a) f_a(k) \). Clearly, the steady-state error is proportional to the magnitude of the disturbance which is a function of the operating speed and load. On the other hand, the steady-state error of the proposed scheme at the \((k+1)\)th instant can be obtained as 

\[
i_a(k+1) - i_a(k+1) = (T/L_a) f_e(k)
\]

where \( e_a = e_q e^T \), \( e_q = f_f f_a \), and \( e_q = f_f f_q \).

Experimental results: The overall system consists of a proposed controller, a disturbance estimator, a PWM inverter, and a PMSM. During the operations, the disturbance estimated by the time delay control is used for the feedforward control. The computed reference voltage is applied using the space vector PWM technique. Based on this, the feedback voltage for the disturbance estimation is obtained from eqns. 7 and 1. In the case of predictive control without feedforward compensation of \( f_a \), the current error at the \((k+1)\)th instant can be derived as 

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Fig. 2 shows the experimental results for the proposed scheme for \( \Delta \omega = -0.5 \). For the time delay, \( L = 1 \) is chosen. Also, for the cutoff frequency of the lowpass filter, \( a = 2000 \) is selected, which corresponds to a cutoff frequency of 318 Hz. Since the rated electrical frequency of the PMSM is 100 Hz and the switching frequency of 7.8 kHz is used, the high frequency noise caused by the inverter switching can be well suppressed without influencing the fundamental current component. Before the estimation algorithm starts, a steady-state error of 0.6 A is observed in the \( q \)-axis current. However, this error is quickly removed within 3 ms as soon as the estimation algorithm starts. The corresponding \( a \)-phase current and the estimated disturbances are shown in Fig. 2b. Fig. 3 shows the control performance under \( \Delta L_2 = 0.8 \). Before the estimation algorithm starts, the phase delay is clearly present in the phase current. Also, some \( d \)-axis current exists (0.5 A), which degrades the performance of the maximum torque operation. However, as the estimation algorithm starts, the \( d \)-axis current is effectively regulated to zero and the phase delay is removed even under such a large variation of \( L_2 \). Fig. 4 shows a comparison of the current responses at starting for \( \Delta \omega = -0.5 \omega_0 \). Under predictive control, the \( q \)-axis current error increases since the magnitude of the disturbances is increased as the rotor speed is increased. However, through the effective estimation of disturbance, the \( q \)-axis current is well controlled to its reference in the proposed scheme.

Conclusions: A simple and robust digital current control scheme for a PMSM using a time delay control approach is presented. The disturbance caused by the parameter variations is estimated by using a time delay control technique. The entire control system is implemented using a TMS320C30 DSP and the effectiveness is verified through the comparative experiments. As a result, the current control performance can be significantly improved by using a relatively simple control algorithm.

References


State estimation via Lyapunov-type fuzzy filter

Neng-Sheng Pai and Tzuu-Hseng S. Li

A novel fuzzy filter is proposed which can be used to solve the state estimation problem. The Lyapunov function is utilized as a performance index to formulate the fuzzy inference rules of the proposed filter. The advantages of this filter are its simplicity, effectiveness and stability.

Introduction: The Kalman filter [1] is well-known for its use in optimal estimation, and is especially suitable for the systems with disturbances and noise. A significant difficulty in designing Kalman filters is how to effectively determine the process noise covariance matrix \( Q \) and the measurement noise covariance matrix \( R \). These matrices are not usually known precisely, or even in a time varying manner. Recently, the use of fuzzy set theory to deal
with the state estimation problem has become of interest [2-4]. Many studies have been carried out into calibrating the covariance matrices $Q$ and $R$ by means of fuzzy rules [4]. In general, the design strategies for the fuzzy decision rules are based on heuristic approaches or on the experiences of human experts. In this Letter, a novel and simple fuzzy estimator is proposed which employs the Lyapunov stability criterion. The proposed algorithm is based on two principle factors. One is the steady state gain matrix of the Kalman filter $K$ and the other is the change rate of the gain matrix $\Delta K$, which is generated by the Lyapunov function and its associated sensitivity function. The obvious advantages of the filter are the lower computational costs and ease of implementation.

Kalman filter: The problem considered here is a linear continuous system, and its dynamic equation is represented as

$$\dot{X}_a(t) = A_x X_a(t) + B_u u(t) + B_v w(t) \tag{1a}$$

$$Y(t) = C_x X_a(t) + v(t) \tag{1b}$$

where $X_a$ and $Y$ are the state, control and measurement vectors, respectively. $A_x$ and $B_u$ are the transition matrices, $C_x$ is the measurement matrix, $v(t)$ is the process disturbance, and $v(t)$ is the measurement noise. Both $v(t)$ and $w(t)$ are assumed to be zero-mean white Gaussian noise with covariance matrices $Q$ and $R$, respectively. Following the standard sampled-data method [5], we can easily obtain the corresponding discrete-time model of the system of eqn. 1:

$$X_a(k + 1) = A_x X_a(k) + B_u u(k) + B_v w(k) \tag{2a}$$

$$Y(k) = C_x X_a(k) + V(k) \tag{2b}$$

where $A_x = \exp [\lambda T A_x]$, $B_u = [\int_0^T A_x B_u]$, $B_v = [\int_0^T A_x B_v]$, $C_x = C_x$, and $T$, is the sampling period. The Kalman filter [1] for the system can be organised as the following two-stage correction:

$$\hat{X}(k|k-1) = A_x \hat{X}(k-1|k-1) + B_u U(k-1) \tag{3}$$

$$P(k|k-1) = A_x P(k-1|k-1) A_x^T + \Gamma W(k) \tag{4}$$

$$K(k) = P(k|k-1) C_x C_x P(k|k-1) + R(k)^{-1} \tag{5}$$

$$\hat{X}(k|k) = \hat{X}(k|k-1) + K(k) (Y(k) - C_x \hat{X}(k|k-1)) \tag{6}$$

Equns. 3 and 4 represent the predicting stage, and eqns. 5-7 represent the updating stage. As we know the Kalman filter is an optimal estimation scheme, which satisfies the mean square error performance criterion. For a linear system, $Q(k)$ and $R(k)$ are always assumed to be constant matrices and determined by the standard deviations of $v(t)$ and $w(t)$, respectively.

Lyapunov-type fuzzy filter (LTFF): First, the proposed Lyapunov function based fuzzy filter for the linear system is organised as follows:

$$\hat{X}_{LT}(k|k-1) = A_x \hat{X}_{LT}(k-1|k-1) + B_u U(k-1) \tag{8}$$

$$\hat{X}_{LF}(k|k) = \hat{X}_{LT}(k|k-1) + K(k) (Y(k) - C_x \hat{X}_{LT}(k|k-1)) \tag{9}$$

where $\hat{X}_{LT}(k|k-1)$ and $\hat{X}_{LF}(k|k)$ are the predicted state and the updated state, respectively, and $K(k)$ is the fuzzy correction gain matrix that is designed not only to guarantee convergence but also to make the estimated state $\hat{X}_{LF}(k|k)$ approach the true state $X(k)$ as soon as possible. Consider the Lyapunov function

$$V(k) = e(k)^T \cdot e(k) \tag{10}$$

where $e(k) = C_x X(k) - X_a(k)$ is the state estimation error vector. The main objective for the proposed fuzzy filter is to determine $K(k)$ such that the Lyapunov difference is guaranteed to be negative, i.e. $\Delta V(k) = V(k) - V(k-1) < 0$.

Let the sensitivity function be defined as

$$S(k) = \frac{\partial V(k)}{\partial K(k)} \nabla V(k) \approx \frac{V(k) - V(k-1)}{K(k) - K(k-1)} = \Delta V(k) \Delta K(k) \tag{11}$$

and

$$\Delta V(k) = \sum_{i,j} \Delta V_i (k) \Delta K_{ij} = \sum_{i,j} \frac{\partial V_i (k)}{\partial K_{ij}} \Delta K_{ij} = \sum_{i,j} \Delta V_i (k) \Delta K_{ij} \tag{12}$$

where $\Delta K_{ij}$ is the $i,j$th entry of the fuzzy gain matrix $K$ and $\Delta K_{ij}$ is the degree of variation to be determined. Here our proposal for designing the fuzzy control scheme is to generate a correct $\Delta K_{ij}$ such that $\Delta V/\Delta K_{ij} \Delta V_{ij} < 0$, i.e. $\Delta V(k)$ will always be negative. In other words, the sign of $\Delta K_{ij}$ is selected to be opposite to that of $S_i$:

$$\text{sign}(\Delta K_{ij}(k+1)) = - \text{sign}(\frac{\partial V_i (k)}{\partial K_{ij}(k)}) = - \text{sign}(S_i(k)) \tag{13}$$

The actual $j$th element of the fuzzy gain matrix is calculated by

$$\dot{K}_{ij}(k+1) = \dot{K}_{ij}(k) + \Delta K_{ij}(k) \tag{14}$$

There is no information about the adjustment of the magnitude $\Delta K_{ij}$ from eqn. 13, although they are useful for determining the proper change of the sign. For this reason we present a method as follows. First, the Lyapunov function $V(k)$ in eqn. 10 can be considered as the distance between the estimated state and the actual state. The main purpose of estimation is to decrease the distance as quickly as possible. Hence, we consider $V(k)$ as an exponential decaying function. To obtain better performance, the hierarchical construction will be exploited. We divide the desired exponential decaying response into three fuzzy subsets, large, medium, and small. Next, the sign of $\Delta V(k)$ indicates whether the state is now diverging from or converging to the actual state $X(k)$. A stronger control action must be taken to drive the divergent states back, and only a medium control command should be required to maintain the movement of the estimated states towards the actual states. When $V(k)$ is small and $\Delta V(k)$ is negative, a smaller control amount is sufficient for obtaining an estimation. From the above discussions, we summarise that the rule base of the fuzzy filter is expressed as

$$\Delta K_{ij}(k+1) = FLD(V(k), \Delta V(k), S(k)) \tag{15}$$

where $FLD(\cdot, \cdot)$ indicates the fuzzy linguistic decision function that is represented in the following form:

- if $V(k)$ is large, $\Delta V(k)$ is negative and $S(k)$ is negative and large, then $\Delta K_{ij}(k+1)$ is positive and small
- if $V(k)$ is small, $\Delta V(k)$ is negative and small, then $\Delta K_{ij}(k+1)$ is small and positive
- if $V(k)$ is small, $\Delta V(k)$ is negative and large, then $\Delta K_{ij}(k+1)$ is negative
- if $V(k)$ is large, $\Delta V(k)$ is negative and small, then $\Delta K_{ij}(k+1)$ is negative

The relative fuzzy rule tables are given in the Table 1. All the rules in Table 1 can be interpreted to be in the same form in eqn. 16. To obtain an estimation, we have to choose an appropriate initial gain matrix $K(0)$. We adopt the steady-state Kalman gain as $K(0)$ since it is independent of the initial guess of $P(0)$. $K(0)$ is calculated by the following equations [1, 3]:

$$A_x P + PA_x^T - P C_x R^{-1} C_x P = - \Gamma Q \tag{17a}$$

$$K(0) = P C_x R^{-1} \tag{17b}$$
The procedure for the proposed fuzzy filter is as follows:

(i) transform the continuous-time linear system of Eqn. 1 into its discrete system, Eqn. 2

(ii) give the initial values of $X(0)$, $\dot{X}(0)$, $Q$, and $R$

(iii) calculate $K(0)$ using Eqn. 17

(iv) calculate the dynamic and measurement equation using Eqn. 2

(v) determine $X(k|k-1)$ via Eqn. 8

(vi) determine Lyapunov function $V(k)$ using Eqn. 10

(vii) determine $\Delta K$ using Eqn. 15 and Table 1

(viii) calculate the fuzzy gain matrix $K$ using Eqn. 14

(ix) calculate the updated estimated state $X(k|k)$ using Eqn. 9

(x) repeat from (iv).

The continuous-time dynamic equation [1] is described as:

$$\dot{x}(t) + 2\zeta \omega_n x(t) + \omega^2 x(t) = 12 + w(t)$$

and its state-variable form is

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\zeta \omega_n \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ 12 \end{bmatrix}$$

The observation equation is $z(t) = x(t) + v(t)$. The initial conditions and parameter values are $x_1(0) = 0$, $x_2(0) = 0$, $Q = 4.77$, $R = 0.01$ and

$$P(0) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

It is shown in Fig. 1 that the estimation trajectory of the fuzzy filter is faster and smoother than that of the Kalman filter. The average results of the 100 MAEs (mean of absolute errors) and the corresponding standard deviations are depicted in Table 2. It is obvious that the fuzzy filter shows a significant improvement in either position or velocity estimations. That is the proposed fuzzy filter not only guarantees the stability for the estimation but also leads to a precise estimation. Moreover, the fuzzy filter gain matrix can be easily calculated by using the fuzzy rule table.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>MAE</th>
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<tbody>
<tr>
<td>Kalman filter</td>
<td>0.0242</td>
<td>0.0310</td>
</tr>
<tr>
<td>Fuzzy filter</td>
<td>0.0181</td>
<td>0.0211</td>
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</tbody>
</table>

References

5. PHILLIPS, C.L., and NAGLE, H.T.: ‘Digital control system analysis and design’ (Prentice-Hall, 1990)